

The influence of cosmic-rays on the magnetorotational instability

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Abstract We present a linear perturbation analysis of the magnetorotational instability in the presence of the cosmic rays. Dynamical effects of the cosmic rays are considered by a fluid description and the diffusion of cosmic rays is only along the magnetic field lines. We show an enhancement in the growth rate of the unstable mode because of the existence of cosmic rays. But as the diffusion of cosmic rays increases, we see that the growth rate decreases. Thus, cosmic rays have a destabilizing role in the magnetorotational instability of the accretion discs.

Keywords galaxies: active - black hole: physics - accretion discs

1 Introduction

Understanding the true nature of accretion processes in astrophysics has always been an attractive research topic over the last three decades. Accretion discs are observed in many astrophysical systems from new born stars to compact objects or even very large discs at the center of the galaxies. In spite of the diversity of the accreting systems, existence of a possible mechanism of the angular momentum transport is a common feature in all these accretion systems. Extensive efforts to understand mechanisms of the angular momentum transport in the accretion discs have lead to a better understanding of such systems, though there are many theoretical and observational uncertainties.

It has been proposed that the magnetorotational instability (MRI) is the main driving mechanism of

turbulence in the accretion discs (Balbus and Hawley 1991). Extensive subsequent works have clarified and extended our understanding of the role of MRI in various astrophysical systems from protoplanetary discs (e.g., Sano and Miyama 1999) to the protostellar discs or even quasar discs. Over recent years a multi layer model for the protoplanetary discs is proposed in which the surface layers are magnetically active due to the ionization of the CRs, while the central layers are magnetically inactive because of inability of CRs to penetrate down to the central parts (e.g., Gammie 2001). Thus, MRI can act as a driving mechanism of the turbulence and the accretion at the surface layer in a protoplanetary disc. However, possible *dynamical* effects of CRs on MRI have not been studied to our knowledge.

Cosmic Rays are very energetic particles but their energy density is in equipartition with energy densities of thermal gas and turbulence (e.g., Ferrière 2001). CRs can act as a source of heating and increase the level of the ionization in the interstellar medium (e.g., Field, Goldsmith, and Habing 1969). An enhanced flux of CRs has important consequences for star formation near to the Galactic center (Yusef-Zadeh, Wardle, and Roy 2007).

However, interaction of CRs with a plasma is not restricted just to a possible enhancement of the level of ionization as have been studied extensively over recent decades. For example, dynamical effect of CRs has a vital role in analysis of Parker instability for the structure formation in the Galaxy at large scale (e.g., Parker 1966; Mouschovias 1974; Hanasz and Lesch 2000; Kuwabara, Nakamura, and Ko 2004; Kuwabara and Ko 2006). It is also found that CRs have a stabilizing effect on the thermal instability (Kuznetsov and Ptuskin 1983; Wagner *et al.* 2005; Shadmehri 2009).

The problem of the diffusion of CRs and its role in MRI has not been studied in detail. Considering CRs as a separate fluid and their diffusion along the magnetic

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field lines, we study MRI in the presence of CRs via a linear perturbation analysis. Our basic equations and the assumptions are presented in the next section. Final dispersion relation will be analyzed in sections 3 and 4.

2 General Formulation

In this study, CRs are protons, electrons and nuclei. But we neglect the electrons because of their little contribution to the total pressure. There are three different approaches to study the dynamics of CRs. In the particle-particle approach, the plasma and CRs are considered as particles that may interact with each other via complicated processes. In a simpler approach, known as fluid-particle, the plasma is treated as a fluid, though CRs are still described as particles. The simplest approach is the fluid-fluid approach in which CRs and the thermal gas are described by different interacting fluids. The hydrodynamic approach can not provide the spectrum of CRs, however, it is a good approximation for analyzing dynamics of a plasma with CRs (e.g., Drury and Voelk 1981; Drury 1983). Thus, we adopt the hydrodynamic approach to study the effects of CRs on the unstable modes of MRI.

We also follow the same steps as in, except that CRs are considered as a fluid, and diffusion is considered only along magnetic field lines. For simplicity, we neglect the ionization and the heating by the CRs, since their effects in the absence of the dynamical role of CRs are well understood.

The basic equations are

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v}, \quad (1)$$

$$\rho \frac{d\mathbf{v}}{dt} = \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla(p + p_{\text{cr}}), \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (4)$$

$$\rho \frac{de}{dt} = -p \nabla \cdot \mathbf{v} + \Lambda, \quad (5)$$

and

$$\frac{1}{\gamma_{\text{cr}} - 1} \frac{dp_{\text{cr}}}{dt} - \frac{\gamma_{\text{cr}}}{\gamma_{\text{cr}} - 1} \frac{p_{\text{cr}}}{\rho} \frac{d\rho}{dt} + \nabla \cdot \mathbf{\Gamma} = 0, \quad (6)$$

where $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ is the Lagrangian time derivative. Here, ρ , p , p_{cr} , \mathbf{v} and \mathbf{B} are the density, gas pressure, cosmic ray pressure, velocity and the magnetic

field, respectively. Also, the net cooling function is denoted by Λ in the energy equation (5). However, we will neglect the net cooling in this paper, i.e. $\Lambda = 0$. Moreover, we have

$$\mathbf{\Gamma} = -\kappa_{\parallel} \mathbf{b}(\mathbf{b} \cdot \nabla p_{\text{cr}}), \quad (7)$$

is the diffusive flux of cosmic-ray energy and κ_{\parallel} is diffusion coefficient along magnetic field lines. All the variables have their usual meaning. Also, \mathbf{b} is a unit vector along the magnetic field lines, i.e. $\mathbf{b} = \mathbf{B}/B$. The adiabatic indices of the thermal gas and cosmic rays are denoted by γ_{g} and γ_{cr} , respectively. Finally, we can write equation of state as

$$p = \frac{R}{\mu} \rho T, \quad (8)$$

where R is the gas constant and μ represents the molecular weight.

3 Linear Perturbations

Assuming that the system is axisymmetric, we write basic equations in the cylindrical coordinates (R, Φ, z) . The equilibrium magnetic field is assumed to be constant in space and only the toroidal and the vertical components of the magnetic field are considered, i.e. $\mathbf{B}_0 = B_z \mathbf{e}_z + B_{\Phi} \mathbf{e}_{\Phi}$. The equilibrium disc is rotating with Keplerian angular velocity, i.e. $\Omega = \sqrt{GM/R^3}$ where M is the mass of the central object. Moreover, the initial density and the gas and the cosmic ray pressures are considered to be constant. Now, we can perturb the equations of the form $\delta X = \delta X_0 \exp i(k_R R + k_z z - \omega t)$. Thus, the linearized equations become

$$-i\omega \frac{\delta \rho}{\rho} + ik_R \delta v_R + ik_z \delta v_z = 0, \quad (9)$$

$$-i\omega \delta v_R - 2\Omega \delta v_{\Phi} + \frac{ik_R}{\rho} \delta p + \frac{ik_R}{\rho} \delta p_{\text{cr}}$$

$$- \frac{ik_z}{4\pi\rho} B_z \delta B_R + \frac{ik_R}{4\pi\rho} (B_{\Phi} \delta B_{\Phi} + B_z \delta B_z) = 0, \quad (10)$$

$$-i\omega \delta v_{\Phi} + \frac{\kappa^2}{2\Omega} \delta v_R - \frac{ik_z}{4\pi\rho} B_z \delta B_{\Phi} = 0, \quad (11)$$

$$-i\omega \delta v_z + \frac{ik_z}{\rho} \delta p + \frac{ik_z}{\rho} \delta p_{\text{cr}} + \frac{ik_z}{4\pi\rho} B_{\theta} \delta B_{\theta} = 0, \quad (12)$$

$$-i\omega \delta B_R - ik_z B_z \delta v_R = 0, \quad (13)$$

$$-i\omega\delta B_\Phi - ik_z B_z \delta v_\Phi - \frac{d\Omega}{d\ln R} \delta B_R + ik_R B_\Phi \delta v_R$$

$$Z^2 c_0^2 \alpha^2 (\gamma_g + \gamma_{cr} \varphi)] \cos \Theta \quad (25)$$

$$+ ik_z B_\Phi \delta v_z = 0, \quad (14)$$

$$P_6 = 2 \gamma_g c_0 \psi_0 q \alpha^2 \Omega^2 Z (\gamma_{cr} - 1) \cos^2 \Theta \quad (26)$$

$$-i\omega\delta B_z + ik_R B_z \delta v_R = 0, \quad (15)$$

$$P_7 = 2 \gamma_g \alpha^2 \Omega^2 \cos^3 \Theta. \quad (27)$$

$$-i\omega \frac{\delta p}{p} + i\omega \gamma_g \frac{\delta \rho}{\rho} = 0, \quad (16)$$

$$\left[\frac{-i\omega}{\gamma_{cr} - 1} + \kappa_{\parallel} (\mathbf{b} \cdot \mathbf{k})^2 \right] \delta p_{cr} + \frac{\gamma_{cr}}{\gamma_{cr} - 1} \frac{p_{cr}}{\rho} i\omega \delta \rho = 0, \quad (17)$$

where κ^2 is the square of the epicyclic frequency,

$$\kappa^2 = \frac{2\Omega}{R} \frac{d}{dR} (R^2 \Omega). \quad (18)$$

Introducing a new dimensionless variable as $X = \omega/(i\Omega)$, our final dispersion equation becomes

$$P_7 X^7 + P_6 X^6 + P_5 X^5 + P_4 X^4 + P_3 X^3$$

$$P_2 X^2 + P_1 X + P_0 = 0, \quad (19)$$

where the coefficients are complicated function of the input parameters. We denote the angle between the vector \mathbf{k} and the component k_z by Θ . Also, a nondimensional wavenumber is defined as $Z = k_z V_{Az}/\Omega$. Thus, the coefficients become

$$P_0 = 2 \gamma_g \Lambda \psi q \alpha^2 \Omega^2 c_0^3 Z^5 (\gamma_{cr} - 1), \quad (20)$$

$$P_1 = 2 \Lambda \alpha^2 \Omega^2 c_0^2 Z^4 (\gamma_g + \gamma_{cr} \phi) \cos \Theta, \quad (21)$$

$$P_2 = 2[(1 + c_0^2 \alpha^2)(\kappa^2/\Omega^2) \cos^2 \Theta + \alpha^2 \Lambda + (1 + 2c_0^2 \alpha^2) Z^2] (\gamma_{cr} - 1) \Omega^2 Z^3 \gamma_g c_0 \psi q \quad (22)$$

$$P_3 = -Z^2 \Omega^2 \cos^2 \Theta \{ \gamma_g c_0 \psi q Z \alpha (\gamma_{cr} - 1) (4 + 2\lambda - \kappa^2/\Omega^2) \sin \Theta - 2 \cos^2 \Theta [c_0^2 \alpha^2 (\gamma_g + \gamma_{cr} \varphi) \kappa^2/\Omega^2 + \gamma_g (2\lambda \alpha^2 + \kappa^2/\Omega^2)] - 2Z^2 [2c_0^2 \alpha^2 (\gamma_g + \gamma_{cr} \varphi) + \gamma_g (1 + \alpha^2)] \} \quad (23)$$

$$P_4 = \gamma_g Z \Omega^2 [\alpha (\kappa^2/\Omega^2 - 2\lambda - 4) Z \cos^2 \Theta \sin \Theta + 2c_0 \times \psi_0 q \alpha^2 (\gamma_{cr} - 1) (Z^2 - \kappa^2/\Omega^2) \cos^2 \Theta + 2c_0 \psi_0 q (\gamma_{cr} - 1) \times (1 + \alpha^2 + c_0^2 \alpha^2) Z^2] \quad (24)$$

$$P_5 = 2\Omega^2 [\gamma_g \alpha^2 (Z^2 + \kappa^2/\Omega^2) \cos^2 \Theta + \gamma_g Z^2 (1 + \alpha^2) +$$

where $\Lambda = 2\lambda \cos^2 \Theta + Z^2$. We verified that the above dispersion equation reduces to the equation for a case without CRs (e.g., see Eq. (17) of Sano and Miyama (1999)). In the above relations, we have $\lambda = -d(\ln \Omega)/d(\ln R)$, $q = [\mathbf{b} \cdot (\mathbf{k}/k)]^2$. Also, the parameter φ represents the ratio of the pressure of CRs to the gas pressure, i.e. $\varphi = p_{cr}/p$. The initial direction of the magnetic field is denoted by $\alpha = B_z/B_\Phi$. The plasma beta parameter is defined by the poloidal field, $\beta_z = 2c_s^2/\gamma_g v_{Az}^2$ where c_s is the sound speed and v_{Az} is the Alfvén velocity, i.e. $v_{Az} = B_z/\sqrt{4\pi\rho}$. Thus, we obtain $c_s = c_0 v_{Az}$, where $c_0 = \sqrt{(\gamma_g/2)\beta_z}$. Also, we have $\psi = \kappa_{\parallel} k/c_s = \psi_0 Z/\cos \Theta$, where the nondimensional diffusion coefficient ψ_0 is

$$\psi_0 = c_0^2 \left(\frac{\tau_c}{\tau_D} \right) \left(\frac{\tau_c}{\tau_d} \right), \quad (28)$$

where the diffusion time scale τ_D , the dynamical time scale τ_d and the sound crossing time scale τ_c are defined as

$$\tau_D = \frac{R^2}{\kappa_{\parallel}}, \tau_d = \frac{1}{\Omega}, \tau_c = \frac{R}{c_s}. \quad (29)$$

4 analysis

Equation (19) describes magnetorotational instability with CRs. Considering complexity of the coefficients of our dispersion equation, it is very unlikely to obtain the roots in analytical closed forms. However, we can solve the equation numerically for a disc with the Keplerian angular velocity. Since our goal is to analyze the unstable perturbations, we restrict our study to the roots with positive imaginary part, i.e. $\text{Im}(\omega) > 0$. If we neglect terms corresponding to the CRs, i.e. $\phi = \psi_0 = 0$, our algebraic dispersion relation (19) reduces to the classical dispersion equation (e.g., Sano and Miyama 1999). Among our input parameters, the effects of CRs are described via two input parameters ϕ and ψ_0 . One can vary these parameters to study how the unstable modes are modified due to the existence of CRs.

Typical influence of CR on the magnetorotational instability are shown in all subsequent plots. Figure 1 shows nondimensional growth rate of the unstable mode, ω/Ω , versus nondimensional wavenumber Z .

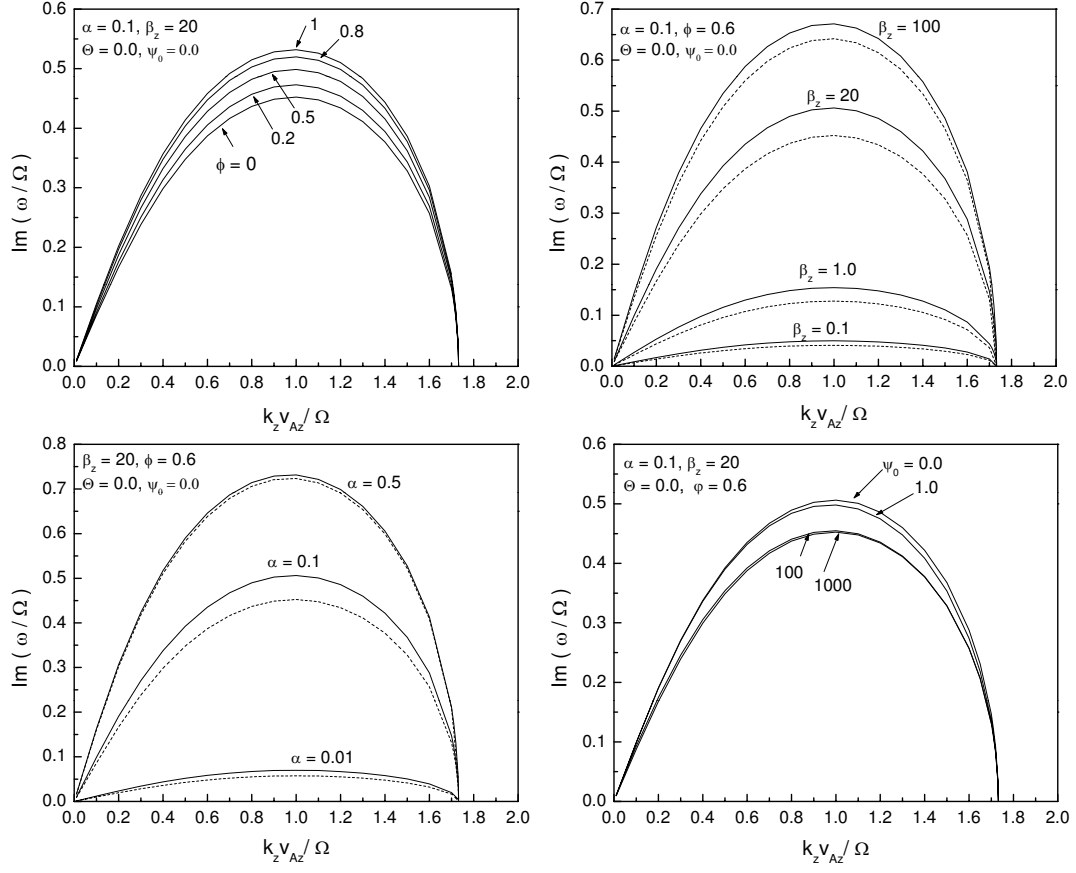


Fig. 1 Imaginary part of the growth rate as a function of the vertical wave number

Each curve is labeled by its corresponding parameter. We found the maximum growth rate occurs for the perturbations with $k_R = 0$ (see also, Sano and Miyama 1999). The top left-hand plot of Figure 1 shows behavior of the unstable perturbation when the ratio of the CRs pressure to the gas pressure φ changes from zero to one with $\psi_0 = 0$, $\Theta = 0$, $\beta_z = 20$ and $\alpha = 0.1$. Here, diffusion of CRs is neglected. Obviously, the case with $\varphi = 0$ corresponds to the unstable mode without CRs. As the ratio φ increases the system becomes more unstable because of the enhancement of the growth rate. In other words, existence of the CRs destabilizes the disc.

The top right-hand plot of Figure 1 shows the typical dependence of the growth rate on the parameter β_z which is ratio of the gas pressure to the magnetic pressure. The input parameters are $\psi_0 = 0$, $\Theta = 0$, $\varphi = 0.6$ and $\alpha = 0.1$ and β_z varies from low value 0.1 to high value 100. Corresponding to each case represented by the solid lines, there is a dashed curve which is for the same case but without CRs. The bottom left-hand plot of Figure 1 shows the case with CRs (solid line) and without CRs (dashed line) for different values of α which is the ratio of the z component of magnetic field to the φ component of the magnetic field. The parameter α changes from low value 0.01 to high value 0.5 and the other input parameters are $\psi_0 = 0$, $\Theta = 0$, $\varphi = 0.6$ and $\beta_z = 20$. In all previous plots, diffusion of CRs along the magnetic field lines is neglected. Now, the bottom right-hand plot of Figure 1 shows how the growth rate of the unstable perturbation is modified when the diffusion of CRs is not negligible. Here, all the input parameters are fixed as $\alpha = 0.1$, $\Theta = 0$, $\varphi = 0.6$ and $\beta_z = 20$ but the dimensionless diffusion parameter ψ_0 changes from 0 to 1000. Diffusion of CRs has a stabilizing effect according to this plot. However, when diffusion is high we can hardly recognize any changes and the plots overlap.

5 conclusion

Our simple approach shows that CRs destabilizes MRI unstable modes. It implies that the generated turbulence because of the MRI would be amplified in the presence of the CRs. However, it seems that growth rate only slightly modifies in the presence of CRs in the linear regime. In our model, gas and CRs coupling is via cosmic ray pressure term in the equation of motion. In other words, CRs provides an extra pressure and we know the pressure plays minor roles in MRI so long as the magnetic pressure is smaller than the gas and the CRs pressures. But when the diffusion of CRs

along the magnetic field lines increases, CRs pressure decreases and the system tends to a case without CRs. Thus, it would be interesting to study MRI with CRs at the nonlinear regime by doing numerical simulations. There are regions with high flux of CRs such as near to the Galactic center or ultra luminous infrared galaxies (Papadopoulos 2010; Papadopoulos *et al.* 2010). Under such circumstances, we think, evolution of the accretion discs located at such regions are significantly affected by the dynamical effects of CRs.

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